

(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 1110

Roll No.

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B. Tech.**(Semester-I) Theory Examination, 2012-13****ENGINEERING MATHEMATICS-I***Time : 3 Hours]**[Maximum Marks 100*

Note : Attempt questions from each Section as per instructions.

Section-A

Attempt *all* parts of this question. Each part carries 2 marks. $2 \times 10 = 20$

1. (a) Write the set of all positive integers whose cube is odd.
- (b) If $A = \{1, 3, 5, 6\}$, $B = \{2, 4\}$, find $A \times B$ and $B \times A$.
- (c) Find the slope of a line which passes through the points $(3, 2)$ and $(-1, 5)$.
- (d) Find the coordinates of the focus and the equation of the directrix of the parabola $y^2 = -12x$.

- (e) Evaluate $\lim_{x \rightarrow 0} x \tan^{-1} \left(\frac{1}{x} \right)$.
- (f) Find the differential coefficient of $(\sin x)^{\log x}$.
- (g) State Rolle's theorem.
- (h) Show that $\frac{1}{x}$ is continuous at $x = a$ ($a \neq 0$).
- (i) If ${}^nP_4 = 12 \times {}^nP_2$, find n .
- (j) In how many ways can a cricket eleven be chosen out of a batch of 15 players, if a particular player is never chosen?

Section-B

Attempt any *three* parts of this question. $10 \times 3 = 30$

2. (a) Find the range of the function $f(x) = \frac{x}{x^2 + 1}$.
- (b) Find the equation of the ellipse whose centre is at the origin, foci are $(1, 0)$, $(-1, 0)$ and eccentricity is $\frac{1}{2}$.
- (c) A function $f(x)$ is defined as follows :
- $$\begin{aligned} f(x) &= \frac{x^2}{a} - a && \text{when } x < a \\ &= 0 && \text{when } x = a \\ &= a - \frac{a^2}{x} && \text{when } x > a, \end{aligned}$$
- where $a \neq 0$. Is the function continuous at $x = a$?
- (d) Differentiate $\left[\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)} \right]$ w.r.t. x .

- (e) A student is allowed to select at most n books from a collection of $(2n+1)$ books. If the total number of ways in which he can select at least one book is 63, find the value of n .

Section-C

Attempt any *two* parts from each question. All questions are compulsory. $5 \times 2 \times 5 = 50$

3. (a) If X and Y are two sets such that X has 40 elements, $X \cup Y$ has 60 elements and $X \cap Y$ has 10 elements. How many elements does Y have?

- (b) Let A and B be sets. Show that :

$$A \cup (A \cap B) = A.$$

- (c) A relation is defined on the set Z of integers as follows :

$$(x, y) \in R \Leftrightarrow x^2 + y^2 = 25.$$

Express R and R^{-1} as the set of ordered pairs and hence find their respective domains.

4. (a) Find the coordinates of the foot of the perpendicular drawn from the point $(1, -2)$ on the line $y = 2x + 1$.
- (b) Find the equation of the circle which passes through the points $(2, -2)$ and $(3, 4)$ and whose centre lies on the line $x + y = 2$.

- (c) Find the equation of the hyperbola whose foci are $(0, \pm 12)$ and the length of the latus rectum is 36.

5. (a) Evaluate $\lim_{x \rightarrow 0} \frac{\sqrt{(1+x)} - 1}{x}$.

- (b) Find the derivative of $f(x)$ from the first principles where $f(x)$ is given by :

$$f(x) = \left(x + \frac{1}{x} \right).$$

- (c) Find the derivative of :

$$f(x) = 1 + x + x^2 + x^3 + \dots x^{50} \text{ at } x = 1.$$

6. (a) Find $\frac{dy}{dx}$, where $x = a \left[\cos t + \log \tan \left(\frac{t}{2} \right) \right]$,
 $y = a \sin t$, a is a constant.

- (b) If $y = A \exp(mx) + B \exp(nx)$, show that :

$$\frac{d^2 y}{dx^2} - (m+n) \frac{dy}{dx} + mny = 0.$$

- (c) If the function defined by :

$$f(x) = \begin{cases} x+5, & \text{if } x \leq 1 \\ x-5, & \text{if } x > 1 \end{cases}$$

a continuous function.

7. (a) If ${}^nP_r = {}^nP_{r+1}$ and ${}^nC_r = {}^nC_{r+1}$, find n and r .

- (b) In how many ways can 5 girls and 3 boys be seated in a row so that no two boys are together?

- (c) If ${}^{15}C_r : {}^{15}C_{r+1} = 11:5$, find 8C_r .